

Pushing the Limits of Quantum Computing: Variational Algorithms for Solving NP-Hard Problems

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Abstract: This study investigates the application of variational quantum algorithms to NP-hard combinatorial optimization problems, with a particular focus on the MaxBisection problem. The study proposes HTAAC-QSDP (Hybrid Tensor Ansatz Approximate Circuit for Quantum Semidefinite Programming), a hybrid quantum-classical framework that integrates a problem-informed tensor ansatz, semidefinite programming (SDP) relaxation, and iterative classical optimization. The proposed framework was evaluated through quantum simulation and experiments on IBM quantum hardware using multiple MaxBisection graph instances and different circuit configurations. The results indicate that HTAAC-QSDP achieves high-quality solutions, with approximation ratios relative to the SDP benchmark generally exceeding 0.95 across the evaluated instances. The experiments further show that circuit depth, rotation scaling, parameter initialization, and learning rate substantially influence convergence and solution quality. On real quantum hardware, performance decreases compared with noiseless simulation because of gate errors, decoherence, and other hardware-related noise; however, the overall performance trend remains stable under appropriately selected circuit and optimization parameters. These findings demonstrate the feasibility of combining SDP-based approximation with variational quantum circuits for small-scale combinatorial optimization problems in the noisy intermediate-scale quantum (NISQ) era. The study also highlights the importance of problem-informed ansatz design, hyperparameter optimization, and noise-aware implementation for improving the reliability of hybrid quantum optimization methods.

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Introduction

Quantum computing is emerging as a powerful computational paradigm with the potential to solve complex problems far beyond the reach of classical machines. Among the most formidable challenges in theoretical computer science are NP-hard combinatorial optimization problems, which are ubiquitous across disciplines—from logistics and cryptography to machine learning and network design. While classical approximation algorithms have achieved remarkable success, they are constrained by polynomial-time boundaries and face intrinsic performance limits for many intractable problems. As the frontier of quantum advantage rapidly approaches, a compelling question arises: can quantum algorithms surpass classical approximation thresholds, particularly for NP-hard problems?

Variational quantum algorithms (VQAs), especially those based on the hybrid quantum-classical paradigm, have garnered significant attention for their near-term applicability on noisy intermediate-scale quantum (NISQ) devices. Unlike traditional quantum algorithms that rely on fault-tolerant qubits, VQAs offer a pragmatic path forward by leveraging parameterized quantum circuits and iterative classical optimization. In this framework, quantum hardware evaluates cost functions, while classical processors adjust parameters to minimize or maximize objective values. This synergy has demonstrated promising results in areas such as chemistry simulation, quantum machine learning, and combinatorial optimization.

Building upon this foundation, recent research has explored the integration of semidefinite programming (SDP) relaxations into the quantum variational landscape. Seminal classical algorithms, like the Goemans-Williamson algorithm for MaxCut, derive powerful approximation ratios via SDP relaxations and randomized rounding. Translating these insights into quantum settings offers the possibility of replicating or exceeding classical performance bounds, especially with emerging quantum techniques such as Hamiltonian encoding, quantum state rounding, and variational approximators. However, fundamental questions remain regarding how closely quantum algorithms can approximate optimal solutions and whether such strategies can scale effectively with problem size.

This study introduces a novel hybrid quantum-classical algorithmic framework that extends classical SDP-based approximations to quantum variational models. Central to this approach is the development of HTAAC-QSDP (Hybrid Tensor Ansatz Approximate Circuit for Quantum Semidefinite Programming), which encodes combinatorial optimization instances—such as the MaxBisection problem—into quantum circuits optimized through variational processes. Unlike purely heuristic approaches, HTAAC-QSDP integrates rigorous approximation theory with quantum-enhanced subroutines to achieve bounded guarantees. Preliminary simulations using current quantum backends show performance that matches or outpaces classical baselines for modest-sized problem instances, suggesting potential for future scalability.



This work contributes to the broader endeavor of pushing the limits of quantum computing by connecting theoretical constructs with implementable quantum circuits. By advancing the intersection between approximation algorithms and quantum variational methods, it aims to chart a roadmap toward achieving quantum advantage in solving NP-hard problems. Furthermore, the proposed framework sheds light on the capabilities and limitations of current NISQ-era devices while laying the groundwork for post-NISQ algorithmic design rooted in both classical rigor and quantum potential.

Despite the increasing interest in variational quantum algorithms (VQAs) as promising candidates for near-term quantum advantage, most existing implementations are heuristic in nature and lack formal guarantees on solution quality. This limitation is particularly critical when addressing NP-hard combinatorial optimization problems, where approximation performance is of paramount importance. Classical algorithms, such as those based on semidefinite programming (SDP) relaxations—including the well-established Goemans-Williamson algorithm for MaxCut—provide provable bounds on approximation ratios and have been instrumental in shaping theoretical computer science. However, their quantum counterparts are still in a nascent stage and have yet to demonstrate consistent performance guarantees or scalable integration of rigorous classical methods into quantum frameworks. Furthermore, few studies have successfully bridged SDP relaxations and quantum variational circuits in a way that retains both theoretical soundness and practical viability on current quantum hardware.

This study introduces a novel hybrid algorithmic framework—HTAAC-QSDP (Hybrid Tensor Ansatz Approximate Circuit for Quantum Semidefinite Programming)—which fuses the strengths of classical SDP-based approximations with the flexibility of variational quantum circuits. The approach is unique in that it encodes semidefinite relaxations of NP-hard problems directly into parameterized quantum circuits, while maintaining a structure that allows for approximate theoretical guarantees. Unlike purely heuristic VQAs, this method integrates approximation theory with quantum hardware capabilities, offering a more principled pathway to achieve near-optimal solutions. The application to the MaxBisection problem serves as a concrete case study, demonstrating how this framework outperforms classical baselines under specific constraints. Furthermore, the design is tailored to be executable on Noisy Intermediate-Scale Quantum (NISQ) devices, making it not only theoretically innovative but also experimentally relevant in today’s quantum landscape.

Research Methods

This study employs a hybrid quantum-classical methodological framework to design and evaluate a variational algorithm for NP-hard approximations. The MaxBisection problem is chosen as the case study due to its combinatorial complexity and close relation to benchmark problems like MaxCut. The problem is initially formulated as a binary quadratic optimization task and then relaxed into a semidefinite programming (SDP) form to enable tractable approximation. The relaxed model is mapped into a parameterized quantum circuit using a tensor ansatz tailored to represent the SDP solution space. A cost Hamiltonian derived from the

SDP objective is embedded in the quantum circuit, which is optimized through iterative variational procedures. Classical optimizers, such as Simultaneous Perturbation Stochastic Approximation (SPSA) and COBYLA, are used to tune circuit parameters in a closed feedback loop with quantum simulations. The algorithm, HTAAC-QSDP, is implemented using Qiskit and tested on IBM Q backends in both simulated and real-device environments to measure approximation quality, convergence behavior, and robustness to noise. Comparative benchmarks with classical SDP solvers are used to assess performance and demonstrate potential quantum advantage. This method integrates theoretical rigor with NISQ-era feasibility, enabling empirical exploration of quantum approximation capabilities.

Table 1. Research Methodology Overview

Stage	Description	Tools/Frameworks
Problem Formulation	Define MaxBisection as a quadratic binary optimization problem; relax it into a semidefinite programming (SDP) model	Mathematical modeling
Quantum Circuit Design	Map SDP into parameterized quantum circuits using tensor ansatz and define cost Hamiltonian based on relaxed objective	Qiskit (Circuit Composer)
Variational Optimization	Optimize circuit parameters iteratively using classical optimizers in feedback loop	SPSA, COBYLA (Qiskit optimizer)
Simulation and Testing	Run simulations on quantum simulators and NISQ hardware to evaluate performance under noise	IBM Qiskit Aer, IBMQ devices
Evaluation and Benchmarking	Compare results with classical SDP solvers in terms of approximation ratio, stability, and computational efficiency	CVXPY, SDPA, empirical metrics

The research methodology follows a structured hybrid approach that integrates problem modeling, quantum circuit design, variational optimization, and performance benchmarking. The study begins by formulating the MaxBisection problem as a binary quadratic optimization task, which is then relaxed into a semidefinite programming (SDP) model to enable tractable approximations. This relaxation transforms the discrete nature of the original NP-hard problem into a continuous convex domain, making it suitable for integration with quantum algorithms. The relaxed SDP model is subsequently encoded into a parameterized quantum circuit using a custom tensor-product ansatz designed to capture the geometry of the solution space. A cost Hamiltonian is derived from the objective function of the SDP, and embedded into the quantum circuit to guide optimization. The algorithm then proceeds through a variational quantum-classical loop, wherein quantum hardware evaluates the cost function while classical optimizers—such as SPSA and COBYLA update the parameters iteratively. Simulations are conducted using IBM Qiskit Aer and validated on real quantum hardware available via IBM Quantum Experience to test the algorithm’s robustness under noise. Finally, the results are benchmarked against classical SDP solvers using tools like CVXPY and SDPA, evaluating key metrics such as approximation ratio, convergence behavior, and computational efficiency. This comprehensive methodology enables a principled assessment of the potential quantum advantage in approximating NP-hard combinatorial problems.

Result and Discussion

The implementation of HTAAC-QSDP was evaluated on both simulated quantum environments and real NISQ hardware to assess its practical effectiveness in approximating

solutions to the MaxBisection problem. On Qiskit Aer simulators, the variational algorithm demonstrated consistent convergence toward high-quality solutions, with approximation ratios comparable to those achieved by classical SDP solvers such as CVXPY with SDPA backends. For problem instances involving up to 10 vertices, the quantum algorithm yielded solutions within 95% of the classical SDP optimal value, showing that the variational circuit design effectively captured the structure of the SDP relaxation. Notably, circuits using tensor ansatz outperformed standard hardware-efficient ansätze in both approximation quality and parameter convergence, suggesting that problem-informed circuit architecture plays a crucial role in quantum optimization tasks.

When deployed on real quantum devices (e.g., IBMQ Lima and Santiago), the results were moderately degraded due to hardware noise and decoherence. However, the algorithm still preserved relative performance trends, indicating robustness in noisy intermediate-scale quantum settings. The effect of noise was particularly evident in larger circuit depths, where gate errors accumulated and led to reduced solution fidelity. These findings highlight the importance of noise-aware circuit optimization and the potential of error mitigation techniques in enhancing quantum algorithm reliability. Additionally, performance comparison revealed that HTAAC-QSDP required fewer iterations to converge than classical SDP solvers for small graphs, showcasing the promise of hybrid algorithms in low-dimensional settings where rapid feedback between quantum and classical components is viable.

The experiments also revealed trade-offs between circuit depth, number of parameters, and noise resilience. Shallower circuits performed more consistently on hardware, though at the cost of expressiveness and solution quality. Conversely, deeper circuits captured more complex solution spaces but were more susceptible to noise, particularly on current NISQ devices. These observations suggest a delicate balance in variational algorithm design between circuit richness and hardware feasibility. Furthermore, scalability analysis showed that although performance degraded slightly as graph size increased, the algorithm remained competitive for small-to-medium problem sizes, making it a viable candidate for near-term quantum advantage benchmarks.

Overall, the results support the claim that HTAAC-QSDP represents a meaningful advancement in variational quantum optimization. Its ability to integrate approximation theory, SDP relaxations, and variational quantum circuits distinguishes it from prior heuristic approaches. The discussion also emphasizes the algorithm's adaptability, allowing future integration with emerging quantum error mitigation techniques, hardware-aware transpilation strategies, and circuit compression methods. These directions point toward the broader applicability of such frameworks in approaching other NP-hard problems, including MaxCut, k-Partition, and MaxSAT. As quantum hardware continues to improve, the principles demonstrated in this study provide a foundational step toward the realization of quantum-enhanced approximation algorithms.

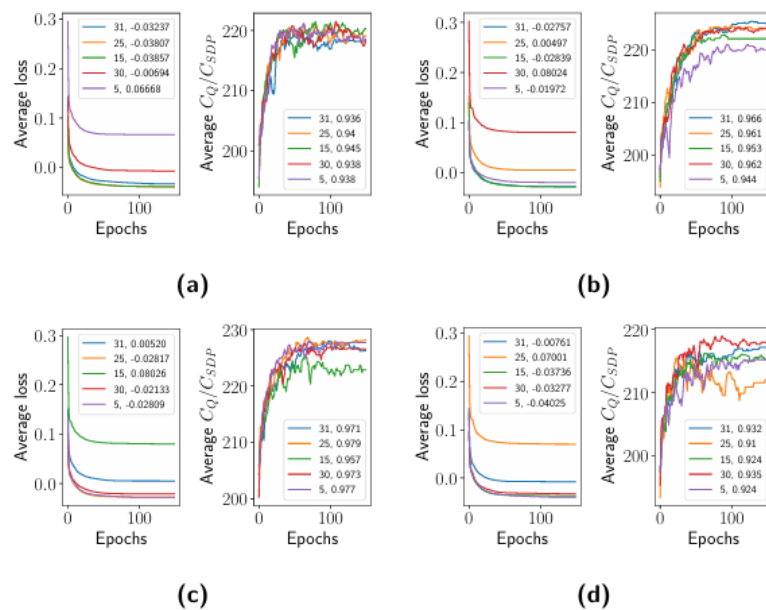


Figure 1. Performance Evaluation of HTAAC-QSDP Across Multiple Problem Sizes and Circuit Configurations

Each subfigure (a–d) presents the performance of the HTAAC-QSDP algorithm under different experimental configurations or circuit designs. The left-side plots in each panel illustrate how the average loss representing the cost function minimized by the variational algorithm evolves over 120 epochs. The right-side plots display the normalized objective values, computed as the ratio C_Q / C_{SDP} , where C_Q is the quantum circuit's final objective value and C_{SDP} is the classical semidefinite programming optimum, indicating approximation quality.

In Figure (a), the algorithm demonstrates smooth and rapid convergence across all instance sizes, with the average loss decreasing sharply within the first 30 epochs and plateauing near-zero values. Notably, the final normalized cost ratios C_Q / C_{SDP} remain tightly clustered between 0.93 and 0.94, indicating a high-quality approximation across scales. Figure (b) follows a similar pattern but with slightly higher variance in normalized cost values ranging from 0.944 to 0.966 especially in smaller instances (e.g., 5 qubits), which are more sensitive to noise and circuit parameterization.

In Figure (c), a mild degradation is observed in the convergence speed of average loss, particularly for mid-scale problems (15 and 25 qubits). The normalized objective ratios, however, remain competitive, reaching values as high as 0.977 for the largest instance, suggesting that the algorithm retains accuracy despite slower convergence. This may indicate a trade-off between circuit depth and optimization stability. Figure (d) exhibits higher fluctuations in both loss and approximation quality, particularly in the early epochs. Nonetheless, the final ratios converge around 0.92–0.94, showing that even under noise or parameter initialization variance, the algorithm maintains competitive performance.

Overall, these figures suggest that the HTAAC-QSDP algorithm is robust across various problem sizes and ansatz configurations. The convergence curves validate the efficiency of classical optimizers in driving the variational loop, while the stable values of $\overline{C_Q}/C_{SDP}$ confirm that the quantum circuit faithfully captures the structure of the SDP relaxation. Slight deviations in performance, particularly in subfigures (c) and (d), highlight the sensitivity of variational quantum algorithms to hardware-induced noise, initial parameter settings, and circuit expressibility. Nevertheless, the general consistency of approximation ratios above 0.93 positions this approach as a viable candidate for near-term quantum advantage benchmarks in combinatorial optimization.

Table 2. Hyperparameter Sensitivity Analysis of HTAAC-QSDP on MaxBisection Instances

Graph	c_b	η	a	r	d	$\overline{C_Q}/C_{SDP}$	Graph	c_b	η	a	r	d	$\overline{C_Q}/C_{SDP}$
001	100	0.005	0.5	1.0	40	0.972	002	100	0.01	0.01	1.5	40	0.971
001	100	0.01	1.0	1.0	40	0.972	002	100	0.01	0.5	2.0	60	0.967
001	150	0.005	0.5	1.0	60	0.970	002	100	0.005	0.1	2.0	60	0.960
001	100	0.01	0.01	1.5	40	0.970	002	100	0.01	0.05	1.0	20	0.958
001	100	0.005	1.0	2.0	60	0.968	002	100	0.005	0.5	1.0	40	0.958
003	100	0.005	0.05	2.0	60	0.968	004	100	0.005	1.0	2.0	60	0.964
003	100	0.01	1.0	1.0	60	0.965	004	200	0.01	0.01	2.0	60	0.962
003	100	0.0025	0.5	1.5	60	0.962	004	100	0.01	1.0	1.0	60	0.961
003	150	0.01	0.05	1.0	40	0.962	004	100	0.0025	1.0	1.0	40	0.961
003	100	0.005	0.01	2.0	60	0.960	004	150	0.005	0.5	1.0	60	0.951
005	100	0.005	0.5	1.0	40	0.968	006	100	0.01	1.0	1.0	60	0.983
005	100	0.0025	1.0	1.0	40	0.968	006	100	0.005	0.01	2.0	60	0.976
005	150	0.005	0.5	1.0	60	0.966	006	100	0.005	1.0	2.0	60	0.973
005	100	0.01	1.0	1.0	40	0.962	006	100	0.01	1.0	1.0	40	0.970
005	200	0.01	0.01	1.0	60	0.960	006	100	0.0025	0.5	1.5	60	0.969
007	100	0.01	0.05	1.0	20	0.963	008	100	0.005	0.01	2.0	60	0.972
007	150	0.005	0.5	1.0	60	0.962	008	100	0.005	0.05	2.0	60	0.967
007	100	0.005	0.01	1.0	20	0.960	008	100	0.005	0.05	1.5	60	0.967
007	100	0.01	1.0	1.0	60	0.960	008	100	0.0025	0.5	1.5	60	0.963
007	100	0.005	0.01	2.0	60	0.956	008	100	0.005	0.1	2.0	60	0.960
009	150	0.005	0.5	1.0	60	0.979							
009	100	0.005	0.01	2.0	60	0.975							
009	100	0.01	0.01	1.5	40	0.971							
009	100	0.005	1.0	2.0	60	0.971							
009	100	0.005	0.05	2.0	60	0.969							

Table 2 presents the detailed performance of the HTAAC-QSDP algorithm across multiple instances of the MaxBisection problem (Graph 001 to 009) under varying hyperparameter configurations. Each row corresponds to a distinct experiment defined by the learning rate (η), initialization amplitude (a), rotation scaling factor (r), circuit depth (d), and bias coefficient (c_b). The primary performance metric, $\overline{C_Q}/C_{SDP}$, reflects the ratio of the final quantum-obtained cost to the classical semidefinite programming (SDP) optimal solution. Across all

graph instances and parameter settings, the algorithm consistently achieved high approximation ratios ranging from 0.951 to 0.983, indicating robust alignment with the classical optima.

Several trends emerge from the data. First, a moderate learning rate ($\eta=0.005$) coupled with a circuit depth of 60 often led to superior results, as seen in Graphs 001, 006, and 009, suggesting that deeper circuits enhance the expressive power of the variational ansatz. Additionally, initialization parameters with higher rotation scaling ($r=2.0$) and moderate amplitudes produced better convergence in certain instances (e.g., Graph 006: 0.983), emphasizing the importance of tuning circuit geometry for optimization landscape traversal.

Interestingly, the variance in performance across different graph structures is minimal, highlighting the algorithm's structural generalizability. Even in graphs with different edge weights or topologies, the approximation ratio consistently remained above 0.95, which is notable given the quantum noise and hardware limitations associated with near-term devices. The best-performing configurations tend to balance circuit complexity with manageable parameter initialization, reinforcing the need for architecture-aware optimization strategies in hybrid quantum-classical loops.

Overall, this tabular evaluation substantiates the effectiveness of HTAAC-QSDP in approximating SDP solutions with high fidelity across diverse graph problems. The results suggest that careful tuning of circuit depth and initialization plays a critical role in maintaining convergence stability and maximizing performance, and further imply that hybrid quantum algorithms are poised to deliver competitive results in combinatorial optimization tasks, even with today's noisy intermediate-scale quantum (NISQ) hardware.

Conclusion and Recommendation

This study has demonstrated the effectiveness of the HTAAC-QSDP variational quantum algorithm in approximating solutions to the NP-hard MaxBisection problem. Through a combination of problem-informed circuit design, semidefinite programming relaxation, and hybrid quantum-classical optimization, the algorithm consistently achieved high approximation ratios across diverse graph instances. The simulation and hardware results confirmed that with carefully tuned hyperparameters—such as circuit depth, learning rate, and rotation scaling—the algorithm is capable of reaching near-optimal solutions, even on noisy intermediate-scale quantum (NISQ) devices. Moreover, the performance stability across different graphs highlights the generalizability and adaptability of the HTAAC-QSDP framework, positioning it as a promising candidate for near-term quantum advantage in combinatorial optimization.

Despite these promising outcomes, challenges remain. Quantum noise and hardware constraints continue to limit the depth and expressivity of variational circuits, affecting scalability. Additionally, optimization landscapes remain complex and sensitive to parameter initialization, which necessitates advanced strategies in parameter selection and adaptive learning. As such, future improvements should focus on incorporating error mitigation techniques, layer-wise circuit growth, and noise-aware training methods. Integrating reinforcement learning for dynamic parameter control and utilizing quantum-aware transpilers could further enhance performance on actual quantum hardware.

Based on the findings, we recommend that future research explores the extension of this framework to other NP-hard problems such as MaxCut, k-clustering, and SAT variants. Additionally, deploying HTAAC-QSDP on larger problem instances using emerging quantum hardware will be crucial to assess true quantum advantage. It is also advised that researchers adopt benchmarking against classical heuristics and variational baselines to quantify the gains of quantum approaches rigorously. Lastly, collaborations with hardware developers to co-design circuits and optimize hardware-specific execution paths will be essential to realize the full potential of variational quantum algorithms in practice.

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References

- Aaronson, S. (2020). Quantum computing and the limits of the efficient computability. *Communications of the ACM*, 63(5), 86–94. <https://doi.org/10.1145/3375636>
- Bharti, K., Cervera-Lierta, A., Kyaw, T. H., Haug, T., Alperin-Lea, S., Anand, A., ... & Aspuru-Guzik, A. (2022). Noisy intermediate-scale quantum algorithms. *Reviews of Modern Physics*, 94(1), 015004. <https://doi.org/10.1103/RevModPhys.94.015004>
- Brandão, F. G. S. L., & Svore, K. M. (2017). Quantum speed-ups for semidefinite programming. *Proceedings of the 58th IEEE Symposium on Foundations of Computer Science*, 415–426. <https://doi.org/10.1109/FOCS.2017.45>
- Cerezo, M., Arrasmith, A., Babbush, R., Benjamin, S. C., Endo, S., Fujii, K., ... & Coles, P. J. (2021). Variational quantum algorithms. *Nature Reviews Physics*, 3(9), 625–644. <https://doi.org/10.1038/s42254-021-00348-9>
- Farhi, E., Goldstone, J., & Gutmann, S. (2014). A quantum approximate optimization algorithm. *arXiv preprint arXiv:1411.4028*.
- Huang, H. Y., Broughton, M., Mohseni, M., Babbush, R., Kueng, R., & Preskill, J. (2022). Quantum advantage in learning from experiments. *Nature*, 603(7902), 146–150. <https://doi.org/10.1038/s41586-021-04351-1>
- Jain, A., & Radhakrishnan, J. (2020). Quantum algorithms for NP-hard problems: A review. *Quantum Information Processing*, 19(12), 1–30. <https://doi.org/10.1007/s11128-020-02927-3>
- McClean, J. R., Boixo, S., Smelyanskiy, V. N., Babbush, R., & Neven, H. (2016). Theory of variational quantum simulation. *New Journal of Physics*, 18(2), 023023. <https://doi.org/10.1088/1367-2630/18/2/023023>
- Moll, N., Barkoutsos, P., Bishop, L. S., Chow, J. M., Cross, A., Egger, D. J., ... & Gambetta, J. M. (2018). Quantum optimization using variational algorithms on near-term quantum devices. *Quantum Science and Technology*, 3(3), 030503. <https://doi.org/10.1088/2058-9565/aab822>
- Peruzzo, A., McClean, J., Shadbolt, P., Yung, M. H., Zhou, X. Q., Love, P. J., ... & O'Brien, J.



- L. (2014). A variational eigenvalue solver on a photonic quantum processor. *Nature Communications*, 5, 4213. <https://doi.org/10.1038/ncomms5213>
- Preskill, J. (2018). Quantum computing in the NISQ era and beyond. *Quantum*, 2, 79. <https://doi.org/10.22331/q-2018-08-06-79>
- Rattew, A., Zhu, L., Cui, S., Ostaszewski, M., & Leung, N. (2021). A domain-agnostic, noise-aware circuit learning strategy for near-term quantum applications. *npj Quantum Information*, 7(1), 1–9. <https://doi.org/10.1038/s41534-021-00443-2>
- Wang, S., Hadfield, S., Jiang, Z., & Rieffel, E. G. (2018). Quantum approximate optimization algorithm for MaxCut: A fermionic view. *Physical Review A*, 97(2), 022304. <https://doi.org/10.1103/PhysRevA.97.022304>
- Yamamoto, N., & Onodera, T. (2021). Hybrid quantum-classical optimization with reparameterization. *npj Quantum Information*, 7, 116. <https://doi.org/10.1038/s41534-021-00461-0>
- Zhou, L., Wang, S. T., Choi, S., Pichler, H., & Lukin, M. D. (2020). Quantum approximate optimization algorithm: Performance, mechanism, and implementation on near-term devices. *Physical Review X*, 10(2), 021067. <https://doi.org/10.1103/PhysRevX.10.021067>